Why We Believe in Quantum Waves

As we discovered last week, quantum mechanics says that every particle is somehow associated with a quantum mechanical wave. There is no statement here what this quantum wave is a wave in, and there is no statement of what the association between the quantum wave and the particle is, except for the following rules:

Where the strength of the particle's associated wave is large, there is some probability that the particle may be found. The greater the strength of the wave at some point, the more likely it is that the particle will be found at that point.

Where the wavelength of the wave is short (or the frequency of the wave is high), any particles found there will have a large amount of motion (momentum, to be exact). Where the wavelength of the wave is long (or the frequency of the wave is low), any particles found there will have a smaller amount of motion.

Just where did the idea of this association come from? It is certainly a strange association, especially when it does not describe the physical status of these waves. These waves are postulated to explain the behavior of very small particles. What is interesting is that it is not at all clear what one has to postulate about these waves except the rules given above. In particular, we do not have to be specific about what these waves are waves in, physically speaking. In fact, it is not at all clear that we have to postulate that these waves actually exist as fundamental entities! It may well be that these waves are only results of some other physical process which we do not yet see. On the other hand it may be that these waves are real fundamental realities. This is the question which is at the heart of the problem of the interpretation of quantum mechanics.

It is hard to imagine two things that are more different than waves and particles. Waves, we are used to thinking, are things that other things **do**. Particles are things that simply exist, at least the most fundamental ones (in fact, this is what fundamental is usually taken to mean!). How can it be that we can confuse them? How can something look like a particle yet be acting like a wave? This is why I have (very carefully) used the phrase "wave associated to the particle". We will soon see, however, that it is difficult to make the distinction between the quantum wave and the particle.

Since these waves are so very important, it is important to be clear as to why we need them to explain particle behavior. While it is true that the wave aspects of particles was first proposed by DeBroglie on theoretical grounds, it is due to observation that we accept them.

There are two particular behaviors of waves that interest us here:

A wave is extended and periodic (i.e. it repeats).

If you add two waves together, you get a single, different wave.

It turns out that we have been able to observe both of these behaviors in the behavior of particles. This is very surprising, as if we were to describe the two behaviors for particles, we would intuitively get something like this:

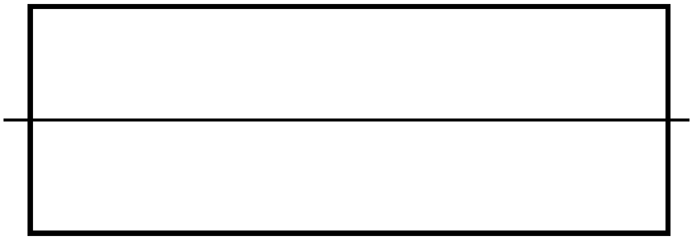
A particle only happens once.

If you add two particles together, you get two particles.

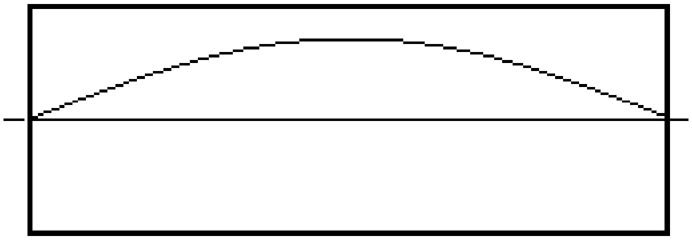
It turns out that the first wave behavior, A wave is extended and periodic (i.e. it repeats), explains the quantization of electron orbits in an atom (and thus atomic spectra). The second wave behavior, If you add two waves together, you get a single, different wave, can be seen directly in what is called an interference experiment.

Atomic Spectra from Quantum Waves

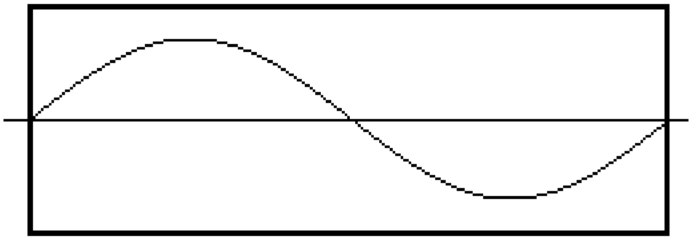
We first look at the motion of an electron around an atom from the quantum wave point of view. Now an electron orbiting an atom is said to be bound to that atom. This is to say that the electron cannot stray too far away from the atom. We can create a simplified picture of this by saying that the electron must be contained within a (very small) box around the atom. Thus inside the box, the electron's quantum wave can have some non-zero strength, but everywhere outside the box the electron's quantum wave must have exactly zero strength. We visualize the box with this figure:



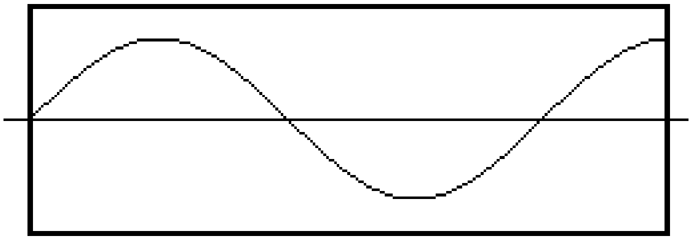
The line through the center of the box represents the zero-strength value of the quantum wave. (We are only considering motion in one dimension here.) Now we can ask what kind of quantum waves are permitted in this box? It turns out that the requirement that the wave vanish outside of the box strongly restricts the possible quantum waves. In particular, if the wave is a simple sine wave, the wave must have zero strength at the edges of the box. Here is the simple sine wave with the longest wavelength that satisfies this criterion:



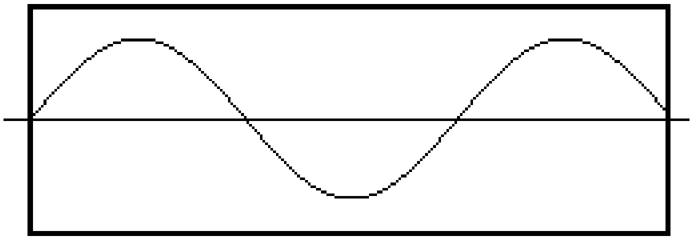
Notice how even though the strength of the wave is non-zero inside the box it is zero at the edges. Now what would the simple sine wave with the next longest wavelength be? This:

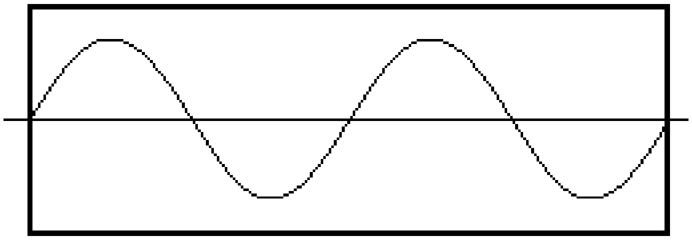


Again, notice how this wave has zero strength at the edges. Here is an example of a wave that would <u>not</u> work:



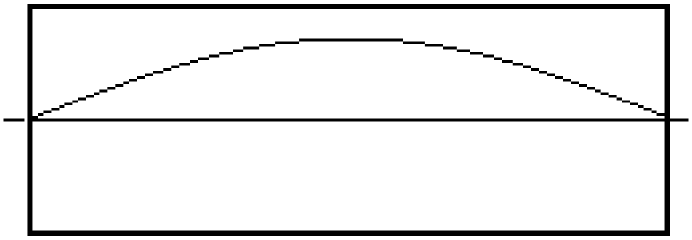
This wave does not work because it is non-zero at the right-hand edge. This violates the idea that the wave is continuous and zero outside the box. Here are some more examples of permitted waves: the next two shorter wavelength waves:





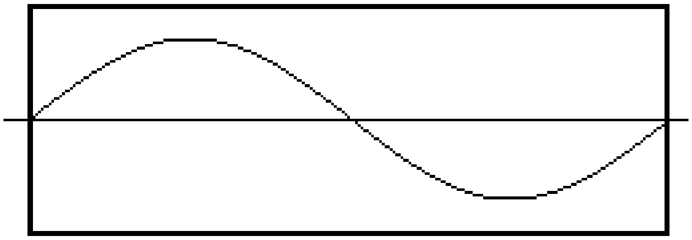
You would continue to get waves of shorter and shorter wavelength as you looked at permitted waves. The important thing to notice that between two successive permitted wavelengths, there are no other permitted wavelengths. Thus when a wave in the box changes from one wavelength to the next shorter one there is a discrete jump of wavelength. The wavelength cannot continually shorten if the wave is to describe the electron around an atom.

Now what are the physical significance of these waves? The wave of the longest wavelength:



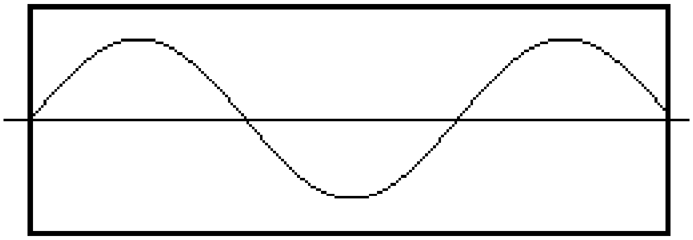
describes the electron in it's state of lowest possible velocity and energy around the atom. This wave indicates that the electron is most likely to be found where the strength of the wave is the greatest, in the center of the atom. Thus the electron in this lowest energy 'orbit' is most likely to be found in the nucleus!

The next lowest energy electron 'orbit' is described by the wave of the next shortest permitted wavelength:

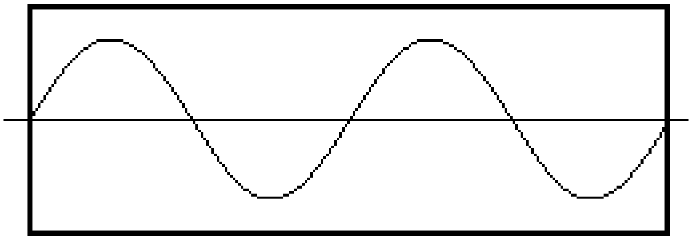


This describes an electron with twice the velocity and four times the energy of the lowest state (energy is given by the square of the velocity). Thus the velocity of the electron must jump from it's lowest value to twice it's lowest value. This is exactly what is seen in atomic spectra.

The next permitted wave is this:



This describes an electron with energy three times that of the lowest energy state (<u>not</u> twice the last state), as it has a wavelength of one third that of the lowest energy state. Similarly, the next wave,

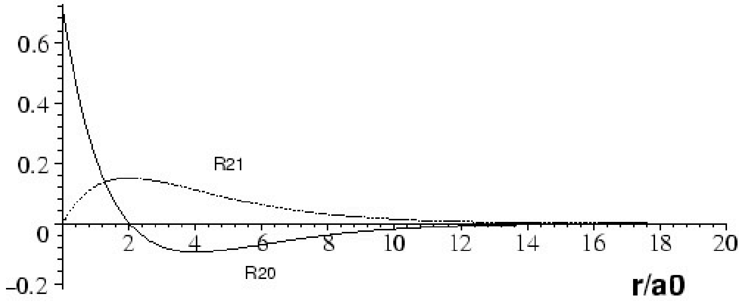


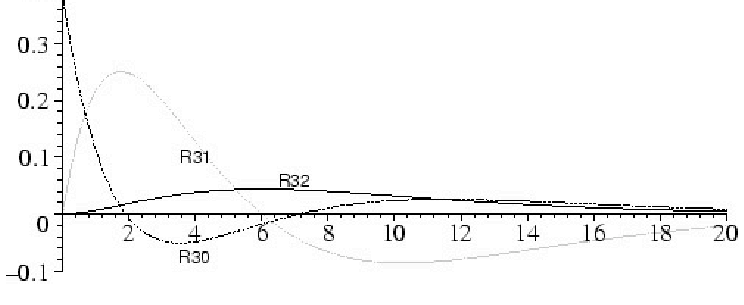
describes an electron with four times the energy of the lowest energy state (why four?).

In this way we see that the demand that the electron be confined to a box constrains the quantum wave in such a way that there only certain discrete possible velocity orbits. First there is the lowest velocity orbit (called the **ground state**), and then only orbits with velocities which are integer multiples of the velocity of the ground state -- 2 times, 3 times, 4 times, 5 times, 6 times and so on. As the multiple gets larger and larger the velocity difference between successive permitted orbits becomes smaller. Thus the second orbit has twice the velocity of the lowest orbit, but the 100th orbit has only a little more velocity than the 99th orbit. This is exactly what is seen in the atomic spectra.

Thus we see that by assuming that a quantum wave is associated with the electron around an atom we come to understand why only certain orbits are possible and we actually correctly predict the relationship of the energies of the orbits. Without the quantum waves this structure of the electron orbits is mysterious.

Here are some examples of real Hydrogen wave functions:





and here is the resulting probability