# Gravity

Gravity is the force of nature of which we are most aware. One can argue that the other forces, such as electromagnetism which holds molecules together in solid objects, or the nuclear forces that determine the structure of atoms, are much more important to us, but somehow they are less obvious. It is the effects of gravity that we readily see, from falling objects to the orbit of the Earth around the Sun. Of all the forces, gravity was studied centuries before the others.

Though we are used to thinking of gravity as a force, it is a force that has some unusual properties:

- We don't feel the force of gravity. While you may think you are feeling gravity when you are, for example, sitting down, what you actually feel is other forces (usually electromagnetic forces) holding you up against the action of gravity. When we are experiencing only gravity (in other words falling) we don't feel any force at all! Astronauts orbiting the Earth are falling in a gravitational field only slightly weaker than the one we live in, but they feel no force at all. (That is why orbiting in space is sometimes called "weightlessness". The technical term used these days is "microgravity", but that is not really correct either. A more correct phrase is "free fall". )
- All objects fall the same (so long as there are no other forces like air friction). This means that gravity acts on all objects in exactly the same way. This property was demonstrated in the famous experiment of dropping different sized balls from the tower of Pisa (not by Galileo, though he verified this fact in other ways).
- Gravity is stronger near massive matter, and the more mass there is the stronger the gravity will be.

With a little thought you'll realize that the first two properties are really the same. If you and your chair were falling in exactly the same way, you would no longer feel the chair holding you up.

Any explanation of gravity has to explain these three properties.

## Isaac Newton's Explanation of Gravity

In the late 17th century, Isaac Newton revolutionized physics by presenting the first integrated, coherent account of motion under the influence of forces. Newton's work included the first quantitative account of gravity.

One of Newton's most important ideas is his law that a force accelerates an object with an acceleration inversely proportional to the mass of that object: for the same force a heavy object will be accelerated less than a light object. This law is summarized as "force equals mass times acceleration" (so acceleration equals force divided by mass). When mass is used in this formula we call is *inertial mass*.

Newton also gave a formula for the gravitational force between two objects: the gravitational force is proportional to the product of the masses of the two objects divided by the square of the distance between them. When mass is used in this formula it is called *gravitational mass*. In this formula Newton explains why gravity is stronger near massive matter, because if the mass is large the force is large, and if the objects are close together the distance is small, so the force is larger.

Newton's formula for gravity creates a mystery, however. If the force of gravity is proportional to the masses of *both* objects, then the force is the different for objects of different masses. Therefore the gravitational force between the Earth and a heavy object is greater than the force between the Earth and a light object. But this seems to contradict the observed property of gravity that both objects will fall the same (ignoring air friction, etc.). The answer lies with Newton's law relating force, mass and acceleration. Remember that the acceleration of a heavy object is inversely proportional to its mass, so it will accelerate slower. This turns out to exactly cancel the action of the stronger force acting on the heavier object, so it will have exactly the same acceleration as the lighter object.

Does this solve the mystery? Not really: the *inertial mass* that appears in the law about force, mass and acceleration is conceptually different from the *gravitational mass* that appears in the formula for the gravitational force. In Newton's theory, these masses have exactly the same value for a particular object. But there is no conceptual reason why these masses should have the same value.

## ***Albert Einstein's Explanation of Gravity***

In the early 20th century, Albert Einstein revolutionized physics in several ways, one of which is the topic of this course. We will spend the first three sessions of the class simply understanding what *General Relativity*, Einstein's explanation of gravity, says. For now, let me just mention the highlights.

- Gravity is not a force, it is objects moving in a *curved spacetime*. When there is no other force involved, objects move along the straightest lines in this curved spacetime.
- Spacetime curvature is caused by matter, and the more massive the matter the greater the curvature.

This is a very abstract idea, not as intuitive as Newton's picture of force and acceleration. But this picture immediately explains the observed properties of gravity:

- Because gravity is not a force, we don't feel it as a force.
- Because objects move along the straightest lines in a curved spacetime (in the absence of other forces), objects move in a way determined by the spacetime they are in, not by the properties of the object. Therefore different objects will fall the same way from the same starting point.
- Curvature is greater near a massive object, so "straightest" lines will be more curved, which will make the effects of gravity stronger.

There are several words in the last paragraph that may be new to you. To understand what Einstein says we have to understand these words: spacetime, curvature, straightest line in a curved spacetime. This determines the agenda for the first half of this course.

Though Einstein's explanation of gravity is very different from Newton's, the predictions are almost the same in everyday gravitational fields. Where the predictions are different, Einstein's are more accurate. In Einstein's explanation there are some very dramatic predictions that have no counterpart in Newton's theory, such as black holes and the possibility that the universe has a shape. There are also less dramatic but important predictions such as gravity waves. These new ideas from general relativity are the topic for the second half of this course.

But first, we can tell the story of general relativity and gravity in a slightly more detailed, but still simple way.

# A Quick Statement of the General Theory of Relativity

## with some history

"*Space acts on matter, telling it how to move.*
*In turn, matter reacts back on space telling it how to curve*"
-- John A. Wheeler

General relativity is, with special relativity, a description of the interaction between spacetime and the matter in that spacetime. General relativity describes the nature of this interaction for any kind of spacetime and special relativity specifies the actual spacetime that we live in. Without postulating any kind of gravitational force, this description predicts that matter will move in a way that almost exactly mimics the action of gravity as described by Newton. This is why we say that General Relativity is a theory of gravity.

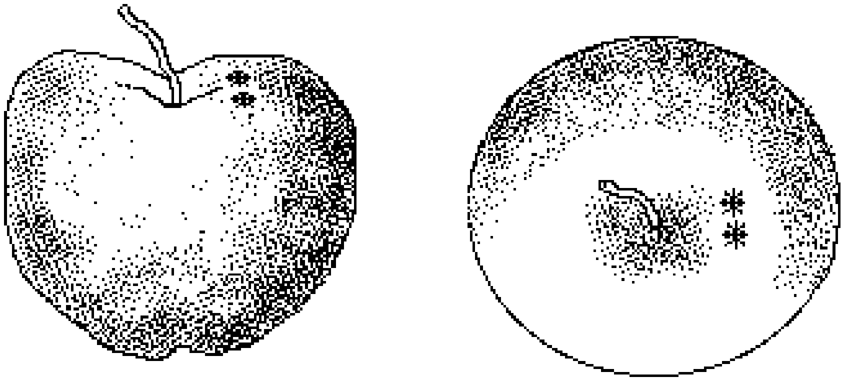
First, what is spacetime (one word!)? For a full treatment, read the intro to space in this handout and attend class next week. For now it suffices to say that spacetime is the space that we live in. Spacetime is, as the word suggests, the wedding of space and time into a larger four-dimensional (three space + one time) spacetime. What we are used to calling "space" and "time" are really different directions in spacetime. You actually know quite a bit about spacetime, having lived in it all your life, but you have been taught to think about space and time as two different kinds of things.

How does matter interact with spacetime and why does this explain gravity? The fundamental concept is that spacetime, like any normal space, can curve. By curve, I mean exactly the same thing as when I say that the surface of an apple is curved. For an apple, the space would be the surface of the apple (a two dimensional space), which is curved. In the same way, spacetime (a four dimensional space) can curve. General relativity says that spacetime is in fact curved in a very specific way by the presence of matter. This is how matter acts on spacetime.

How does curved spacetime act on matter? The curvature of the spacetime will naturally curve any (especially otherwise straight) paths in that spacetime. It is this curving of paths that looks like gravity. To see how the curvature of spacetime can look like gravity, it helps to return to the apple analogy and consider

## ***The Parable of the Ants and the Apple***

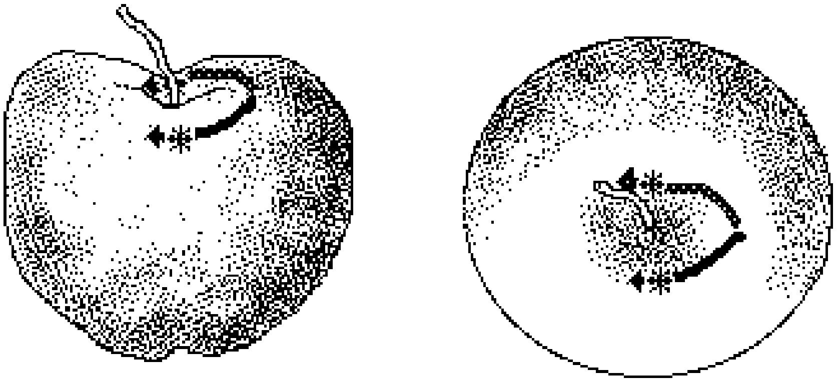
Imagine two ants on the top of an apple not too far from the stem. In the course of their travels they look ahead and see the stem of the apple and they have some disagreement about the best way to proceed.



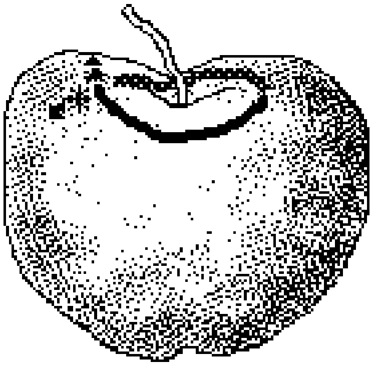
One thinks that going to the left of the stem is the best course and the other thinks that going to the right of the stem would be best. One cannot convince the other of the best course, so they each decide to each go the way he thinks best, with each traveling only forward in order to see who goes where. So they start off in two slightly different directions, walking exactly forward just inside the dimple that contains the apple's stem.

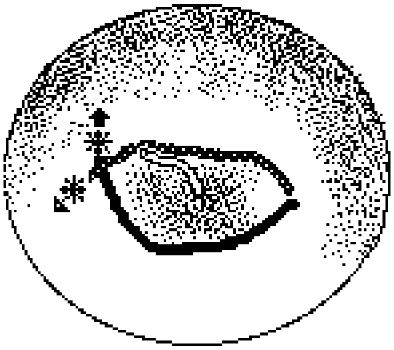
Now at this point you should stop and try to predict what will happen. If the ants were walking on a flat tabletop, they would move further and further apart if they started walking in two different directions. Is that what will happen in this case?

As they walk on, even though each ant is walking exactly forward, not turning left or right, each ant finds that his path is curving around the dimple:



As they continue on, their paths will curve around the stem of the apple and they will meet on the other side!





This, of course, is due to the curvature of the surface of the apple around the stem. Even though each ant was walking exactly forward and did not turn, his path curved around the stem.

Now ants are very short-sighted creatures and cannot see very far. To them the apple is so large that they see only a very small part of the apple at any time. In fact they see so little of the apple that they do not realize that the apple's surface is curved at all (Does this sound familiar? Didn't you have to be told that the surface of the Earth was curved?).

If the ants did not know that the surface of the apple was curved, how would they account for the fact that their paths curved even though they themselves did not turn? Being rather intelligent ants, they would perhaps postulate a force that drew then toward the stem. If and when they developed language they might call this force gravity. They would perform experiments and observe both big and small ants walking straight ahead from the same spot in the same direction follow the same path, and conclude that this force from the stem acts in the same way on all ants regardless of size.

Here is a physicist’s view of the ant’s motion:

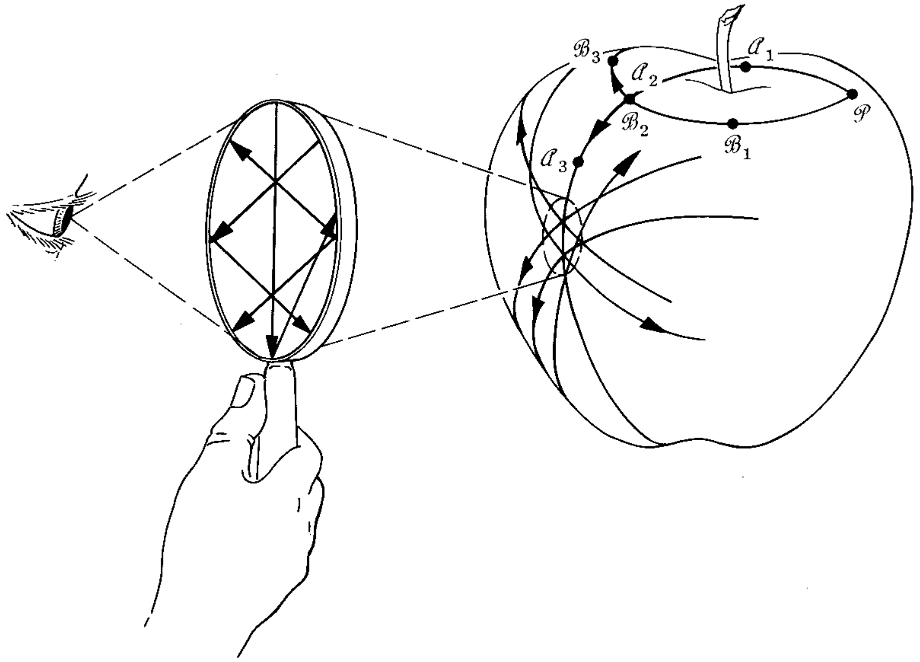
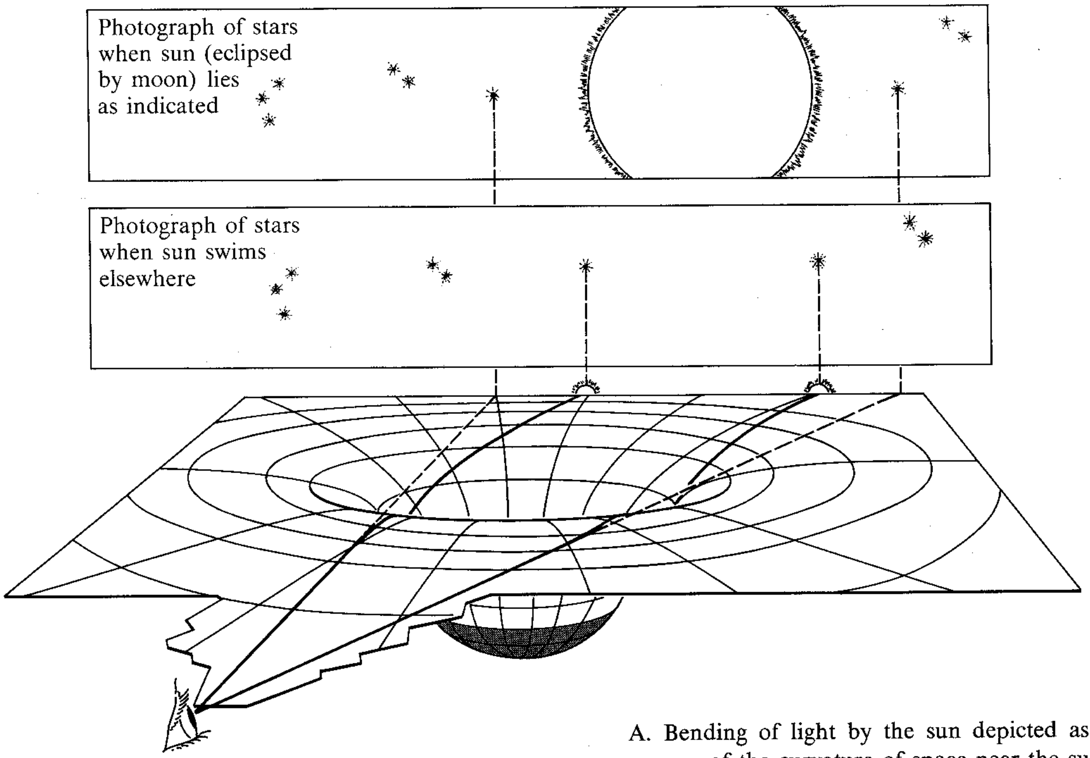


**Figure 1.1.**
The Riemannian geometry of the spacetime of general relativity is here symbolized by the two-dimensional geometry of the surface of an apple. The geodesic tracks followed by the ants on the apple's surface symbolize the world line followed through spacetime by a free particle. In any sufficiently localized region of spacetime, the geometry can be idealized as flat, as symbolized on the apple's two-dimensional surface by the straight-line course of the tracks viewed in the magnifying glass ("local Lorentz character" of geometry of spacetime). In a region of greater extension, the curvature of the manifold (four-dimensional spacetime in the case of the real physical world; curved two-dimensional geometry in the case of the apple) makes itself felt. Two tracks $\mathcal{A}$ and $\mathcal{B}$, originally diverging from a common point $\mathcal{P}$, later approach, cross, and go off in very different directions. In Newtonian theory this effect is ascribed to gravitation acting at a distance from a center of attraction, symbolized here by the stem of the apple. According to Einstein a particle gets its moving orders locally, from the geometry of spacetime right where it is. Its instructions are simple: to follow the straightest possible track (geodesic). Physics is as simple as it could be locally. Only because spacetime is curved in the large do the tracks cross. Geometrodynamics, in brief, is a double story of the effect of geometry on matter (causing originally divergent geodesics to cross) and the effect of matter on geometry (bending of spacetime initiated by concentration of mass, symbolized by effect of stem on nearby surface of apple).

A technical version of "the ants and the apple". From *Gravitation* by Misner, Thorne and Wheeler

In exactly the same way, if spacetime is curved then traveling forward in spacetime (we'll see exactly what this means next week) will in general take us along curved paths. Not just any curved path, however, but along the straightest possible path in that curved space. This is important because in the absence of any forces things move in the straightest possible lines even in a curved space.

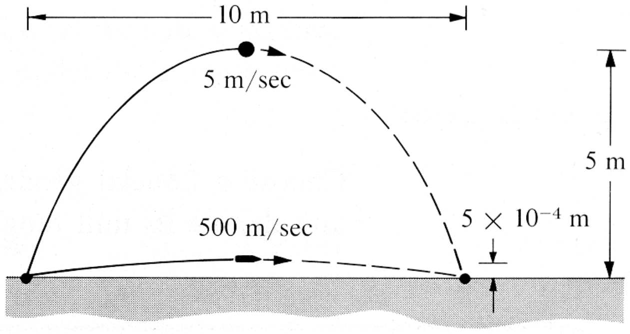
For example, the Earth moves in an approximately elliptical path around the sun. This is not due to the force of gravity (we now know), but is the earth simply following the straightest path it can around the Sun. This is due to the curvature of spacetime around the Sun. (Compare this to the statement about the path of the ants from above that "This, of course, is due to the curvature of the surface of the apple around the stem." The analogy is a precise one.)



A. Bending of light by the sun depicted as a consequence of the curvature of space near the sun. Ray of light pursues geodesic, but geometry in which it travels is curved (actual travel takes place in spacetime rather than space; correct deflection is twice that given by above elementary picture). Deflection inversely proportional to angular separation between star and center of sun. See Box 40.1 for actual deflections observed at time of an eclipse.

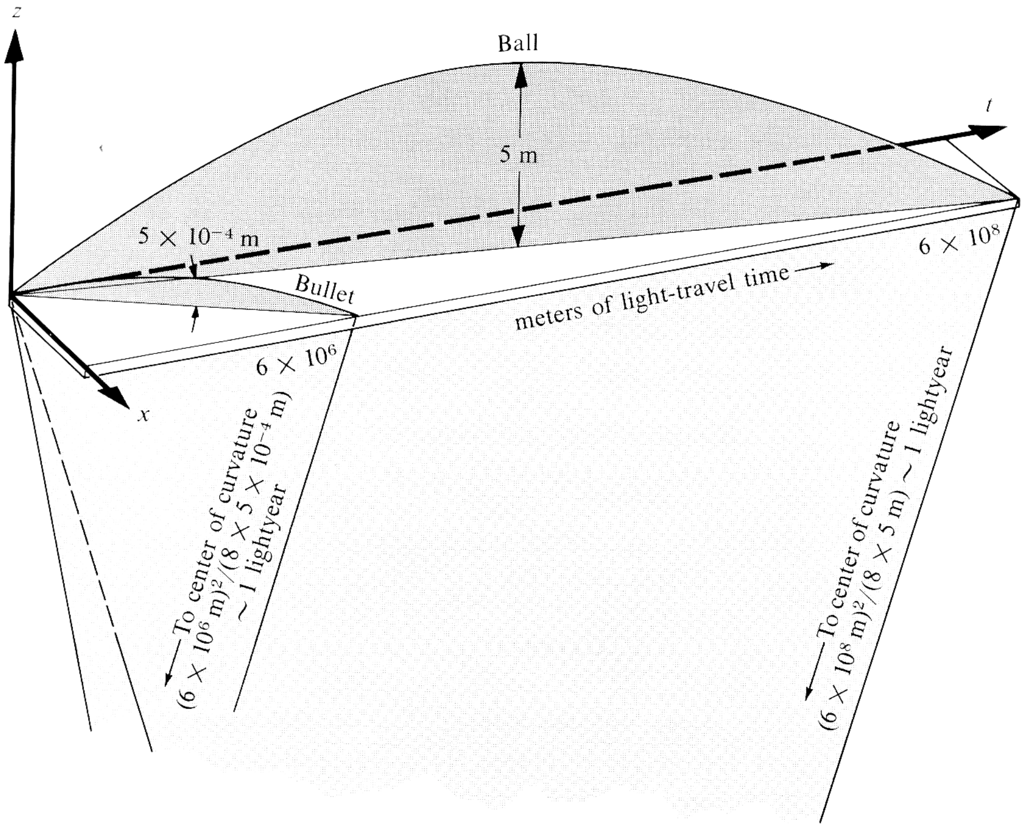
Curvature in spacetime around the Sun. From *Gravitation* by Misner, Thorne and Wheeler

The spacetime around the Sun is curved by the matter inside the Sun, and matter around the sun (Earth, for example) moves in the straightest possible lines in that curvature. It is the absence of any other forces that allows the Earth to move along it's path. In this view, there is no such thing as the 'force' of gravity! This force has been completely replaced by the curvature of spacetime. (We still refer to this curvature as gravity.)



B. Tracks of ball and bullet through space as seen in laboratory have very different curvatures.

The curvature of the paths of a ball and a bullet starting from the same place but with different velocities as seen in space. The initial directions are set so the ball and bullet end up at the same place. These paths look like they have very different curvatures. From *Gravitation* by Misner, Thorne and Wheeler



C. Tracks of ball and bullet through spacetime, as recorded in laboratory, have comparable curvatures. Track compared to arc of circle: (radius) = (horizontal distance)$^2$/8 (rise).

From a spacetime perspective the paths have the same curvature. From *Gravitation* by Misner, Thorne and Wheeler

Now you may object that you certainly experience a force of gravity, all the time! But think about this force you experience. When do you experience it? Only when some other force (like the surface of your chair) besides gravity is holding you up, preventing you from falling! If we were to remove all the other forces, you would start falling.

Try to imagine such a state. Would you actually feel any forces then? No! In fact, if you were falling in an enclosed box and could not see out you would not be able to tell if you were falling due to gravity or were simply transported into deep space far from any source of gravity, where one could say that there was no noticeable gravity. Thus falling is really the experience of not feeling a gravitational force. What you have been taught to think of as the experience of the force of gravity is really the experience of other forces holding you up.

In this way we come to understand how curvature can determine the motion of matter. How is it that matter determines the curvature of spacetime? Though there are some tantalizing hints, we do not understand why matter curves spacetime. But we do know how matter curves spacetime, in the sense that we know how to figure out how a specific bunch of matter will produce a specific spacetime curvature. This is given by what is called 'the field equation of gravitation' and was published by Einstein in November of 1915. Just for fun, here it is (in suitable units):

$$R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R = -8\pi T_{\mu\nu}$$

or:

*Something about curvature = the amount of matter and energy*

Later in the course we'll use pictures to understand what each of these terms mean.

How can we be so sure of all this? As I mentioned early in this handout, motion of matter according to general relativity almost exactly mimics the motion of matter under the gravitational forces as described by Newton. Almost, but not quite exactly. There are small differences between general relativity and Newtonian gravity. Though it is very difficult to do, we can look to see if the real world behaves as predicted by Newtonian gravity or general relativity. In every test performed so far, Newtonian gravity has failed and general relativity made the correct prediction. This is not to say that general relativity has been proven correct (this never happens in physics) but we are becoming very confident that general relativity is an accurate description of the real world. If general relativity is proved someday to be incorrect, whatever replaces it will have to look a whole lot like general relativity, at least in terms of its predictions. My personal prejudice is that general relativity is too pretty to be in doubt.

## *A Little Bit of History*

The history of general relativity is rather unique in the history of physics, because it is a revolutionary theory that was not the product of a crisis in the physics community. It was, however, the product of a crisis for Einstein and he resolved this crisis more or less alone. Here is a chronology of the conceptual and physical developments in general relativity.

First, some mathematicians created the context for, and to some extent anticipated, general relativity, by thinking about the curvature of space.

**1827** **Karl Friedrich Gauss** develops the mathematics of defining the curvature of a two dimensional surface only in terms of measurements performed on that surface. In other words, you need not see the surface from the 'outside' in order to describe it's curvature.

**1851** **Georg Friedrich Bernhard Riemann** generalizes Gauss' mathematics to spaces of arbitrary dimension. Riemann goes on to suggest that the motion of objects due to gravity and electromagnetism could perhaps be understood as motion in a curved space. For the rest of his life Riemann attempts to explain all motion this way. He fails because he is working in the framework of *three-dimensional* space. The required understanding we are really in a *four-dimensional* spacetime was not to come for another 54 years.

**1870** **William Kingdon Clifford**, following Riemann, writes to the Cambridge Philosophical Society "That this variation of the curvature of space is what really happens in that phenomenon which we call the motion of matter, whether ponderable or etherial." He also writes "That in the physical world nothing else takes place but this variation, subject (possibly) to the laws of continuity."

**1879** **Albert Einstein** born.

Riemann and Clifford thus completely anticipate the conceptual content of general relativity. Both Riemann and Clifford are mathematicians with an interest in physics. Both made tremendously significant contributions to other fields of mathematics, Riemann somewhat more so than Clifford. Both died young, Riemann at age 39 and Clifford at age 34. It is fairly clear that if either had understood the four-dimensional nature of spacetime (which Einstein taught us in his theory of special relativity in 1905), we would have had general relativity in the middle of the 19th century, without the conceptual confusions that Einstein went through.

The mathematics invented by Riemann is subsequently developed through the end of the 19th century and the beginning of the 20th by **Gregorio Ricci-Curbastro** (1853-1925) and **Tullio Levi-Civita** (1873-1941). This mathematics comes to be known as **Tensor Calculus**.

**1905** **Albert Einstein** publishes the special theory of relativity in its complete form, after thinking hard about electromagnetism and motion. In this quest, Einstein is anticipated and aided by **Henri Poincare**, **Hendrich Lorentz**, and **George FitzGerald**.

**1907** **Einstein** realizes that Newtonian gravity is incompatible with special relativity. He proposes a primitive version of the equivalence principle ('gravity is just like acceleration'), and derives from it and special relativity the gravitational redshift and the bending of light by gravity (both conceptually correct results but numerically wrong).

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| **1908** | **Hermann Minkowski** elucidates special relativity as being a description of the world as a four dimensional spacetime. |
| **1911** | **Einstein** rewrites the 1907 paper with better arguments and the same (numerically incorrect) results. This paper gets some attention. |
| **1912** | Amid much confusion, **Einstein** begins to catch on that he should be working in a curved spacetime. |
| **1913** | **Einstein** begins to learn the tensor calculus with the help of longtime friend and mathematician Marcel Grossmann. They immediately begin to apply this mathematics to four dimensional spacetime. |
| **1913-15** | **Einstein** writes down many theories of gravity using tensor calculus applied to spacetime. All are wrong, usually because of subtle confusion and misunderstanding of the mathematics. Einstein is working more or less alone during this period, partly because he was perhaps the only physicist trying to work with the tensor calculus. |
| **Nov. 1915** | **Einstein** suddenly becomes unconfused about the proper use of the tensor calculus and announces the correct equation on November 25, 1915. As part of this paper, Einstein correctly predicts the bending of light by the sun, the gravitational redshift, and a previously unexplained anomaly in the orbit of the planet Mercury. |

In 1915 **David Hilbert** (who was the greatest mathematician of his day) joins Einstein in a quest for the correct equations for gravity as the curvature of spacetime. Hilbert publishes the correct equations in a paper that appears in March 1916 but is dated as submitted on November 20 1915, 5 days before Einstein announces his equations. We know that in November 1915 Einstein and Hilbert were writing each other daily about these equations, but around November 20 something happened which caused them to cease communication with a “certain resentment” (Einstein’s words in a letter to Hilbert on December 20, where he tries to restart the relationship). This has led to speculation that Hilbert communicated the correct equations to Einstein around November 18 (which letter, sadly, no longer exists). Some of this speculation (including a previous version of these notes) considers the possibility that Einstein actually plagiarized the correct equations from Hilbert, though most authors conclude that Einstein and Hilbert discovered the equations independently. In this account, the resentment is by Hilbert, thinking that Einstein had not given him appropriate credit.

This question has apparently been recently resolved by the discovery of a galley proof of Hilbert’s March 1916 article. The proofs are dated December 6 1915, and contain several of the same conceptual confusions that Einstein was struggling with in the first half of 1915. In addition, the correct equations do not appear at all in these proofs, and were apparently added later (including resolution of the confusions), between late December and March, presumably based on Einstein’s announced results. The “resentment” is now more likely to be because Einstein felt that Hilbert was not giving sufficient credit to Einstein. Most importantly, though, there can now be little doubt that Einstein came up with the correct equations first.

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| **1917** | **Einstein** writes down first cosmological theory based on general relativity. Here he finds that according to general relativity the universe should either be expanding or contracting. Einstein modifies the field equations to keep the universe from doing this. |
| **1919** | The deflection of light by the sun is detected and is in good agreement with the value Einstein predicted in 1915. This makes Einstein very famous. |
| **1922** | **Aleksandr Friedmann** produces an improved cosmological theory (which is still in vogue today) which predicts that the universe should be either expanding or contracting according to the original field equation. Einstein initially thinks Friedmann is mistaken, but later calls this result "clarifying". |
| **1929** | **Edwin Hubble** discovers that the universe is in fact expanding. |
| **1931** | **Einstein** officially goes back to the original field equations of 1915, calling his modifications of 1917 "the greatest mistake of my life". |
| **1920-59** | General relativity is a removed, highly specialized, difficult and obscure part of physics. Perceived as having no real observational predictions and using an arcane and difficult branch of mathematics, it is practiced by only a handful of physicists. |
| **today** | Largely due to the discovery of exotic astronomical objects such as quasars, black holes, and remnants of the 'big bang', general relativity has experienced a major renaissance. Due to vastly improved and streamlined mathematical techniques, general relativity is far more accessible and is much better understood. It is practiced by a large number of physicists, and is being combined with other theories in physics. |

# Space

## *What is a space? Is it something or is it nothing?*

We are going to be developing a new view of the world. This new view is founded on the idea that space and time combine to form a four-dimensional space that we call spacetime. Before we can do this we need to know what a space is.

The word 'space' is used many different ways in our culture. We speak of empty space, outer space, parking space, and breathing space. If you are from California you may speak of your own personal space and your mind space. The mathematician speaks of abstract spaces while the physicist speaks of configuration spaces. All of these uses of the word 'space' refer to very different things. All of these uses of the word space are completely valid.

Of all the above uses of the word 'space', the use that is closest to the one we shall use is perhaps 'parking space'. It is the place where something happens, namely the parking of a car. It is sometimes empty and sometimes full. Kids can play catch in this space. The space can have properties, like parking regulations, that effect what happens in the space.

In this class, the word space will denote the setting of our world. It is where things happen. People often picture space by picturing an empty container. In this picture, space is, in some sense, what is left over if you remove everything. In the way that I am using the word 'space', this is partly right and partly misleading. It is right in that it shows space to be the background in which things move. It is misleading in that the space that I am referring to is not defined by some container. The space I am talking about is the setting for the entire universe. This picture is also misleading because the empty space is seen to be a passive background in which things happen. We shall see that the space in which everything sits can be a very active participant in what happens in that space.

So what are you supposed to picture when I say space? Think this: Space is the place where things are. This is not a complete definition, but it is enough to get started. We will be getting more precise as we go on. The fuller picture will come as we journey into different kinds of spaces.

A simple example of the kind of space that we will be talking about is called for. Allow me to present you with a simple hand held space: The page that these words are written on. If you look above where you are reading you will see that the space contains many many black marks. For contrast, here is some of the space without any marks, some empty space:

You can, if you wish, put more dark markings in this space. If you are ambitious you can even make markings that move around.

Now what about everything else, like your nose or the streetlight outside my window? Are these things in this paper space? No! Only those things on the paper are in the space of this sheet of paper.

How about another example? A different space is the room that you are now in (this may be a very big space indeed if you are outside). Look around you and name some of the things that are in this space. Things can move in this space. You can throw this paper across the room, you can observe some creature navigating about, you can even move in this space yourself!

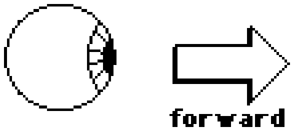
This brings us to the next concept that we must get clear about. What does it mean to move in a space? This is not hard to understand, as we certainly know what it is to move around the room. We need to be precise about this intuition so that we can talk of motion in other spaces. Motion is simply change of position in a space (we will worry about making the concept of position precise later). You move by going from one place to another in a space. If this is too vague for you, it is enough now to think this: motion is going from where you were to where you are.

When you move, you move in a particular direction. How many possible directions can you move in? Infinitely many! You can move forward, sideways, up, and any combination of these three. In the example space of the sheet of paper, a point could also move infinitely many directions, towards the sides of the paper, towards the top, or any combination of the two.
In both the space of your room and the space of the paper there is an infinity of possible directions of motion. Yet your intuition probably says that you have more directions to move in you room than a point has on the sheet of paper. This intuition is correct, and understanding this intuition in a precise way brings us to what is one of the most important properties of a space, its *dimension.*

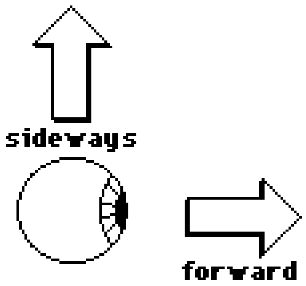
## *The dimension of a space*

In the space of your room, you can choose from an infinity of directions in which to move. Picture yourself now choosing one of those directions. Turn to face in that direction and start to move forward. As you move forward, notice that you are not moving sideways -- that is you are not moving to your right or to your left, at least if you followed my instructions. Notice also that you are not moving up or down. Your nose stays (more or less) at the same altitude from the floor. This should not surprise you because, after all, this is what it means to move forward! But now imagine a direction which is some combination of forward and left. You might point it out by raising your arm in that direction. Now ask yourself "am I moving in the direction of a little bit forward and a little bit left?" You are not moving exactly in that direction, but you are moving a little bit in that direction, as that direction is a combination of forward and left and you are moving forward. Now try to picture other such combinations of forward and sideways or forward and up and down. You are moving a little bit in all these directions if you are moving forward. (Something to think about: are you moving in a direction which is a combination of sideways and up and down only?) However, when you are moving forward you are not moving at all sideways or up and down. Close your eyes, get a good picture of this, and go on.

Now let's pretend we live in the paper space. So that we have a concept of forward in this paper world, let's pretend that we look like a circle with one eye.



When we move in the direction that the eye is pointing, we say that we are going forward. Now when we are going forward, we are not going to our right or left, just like walking around in our room.



Just like in the space of our room, we can choose a direction which is a combination of forward and sideways. When we are moving forward, we are moving a little bit in this new direction. There is one thing about this situation that is different, though, and that is that there is no up and down in the paper world. There is only forwards and sideways.

This is the essential difference between the paper space and the space of your room. In your room when you were moving forward there were two directions that you were not moving in: sideways and up and down. In the paper space, when you move forward, there is only one direction that you do not move in: sideways. This is because the space of your room is a **three-dimensional space** and the space of the paper is a **two-dimensional space**. The idea here is that a space has some special directions. These directions are special in that if you move along one of these directions you do not move along any of the other directions. The dimension of a space is the number of these special directions. Here is the definition of the dimension of a space:

*The dimension of a space is the number of directions in that space so that if you move along one of these special directions you do not move along any of the other special directions.*

There is one subtlety here. The dimension of a space counts the number of directions that are special in the way described above. In the space of your room, that number is three and in the space of the paper that number is two. This just means that, in the space of your room, there are three special directions. **Nothing says which three directions they are.** After all, you choose what forward is. Once you choose what direction forward will be, then the other two special directions are limited: they cannot include any forward component. You can then choose up from this limited set, which fixes sideways. Once you choose up and forward, you have no choice about which direction is sideways. It is important to understand that you choose the three special directions by choosing one of them, which limits the possibilities for the other two, then if you choose the second this determines the third. If you are lying in bed, the your forward would be towards the ceiling, your sideways may be to your left and right, in which case your up and down is along the length of the bed. To say a space is three-dimensional is to say that there three special directions of motion--not which three directions they are.

## Coordinates

Why do we care about the dimension of a space? The real answer is that by understanding the dimension of a space we get deep understanding of how things are in that space. This is very abstract, however. Is there not a more tangible excuse for understanding dimension than this?

Of course there is! We need to know the number of dimensions of a space in order to locate objects in that space. Consider yourself in the space of your room. Holding very still, picture a point in space about two feet in front of your nose. How would you tell your friend, who will be there tomorrow while you are away, where that spot in space? Think about this problem a little before you go on. There are actually many ways to specify where the spot is in space. The most natural way is to say that the spot is so far from the floor, so far from this wall, and so far from this side wall. Notice that you needed three numbers to tell where this point is. You may specify the point by saying something like "It is two feet in front of me." This seems to only use two numbers (distance and direction). Actually, however, for this to work you need to specify your position in space and which direction you are facing, and that will take at least three numbers. You can be very clever and try to come up with other ways to specify the position of that point in this space. If you do this correctly, you will find that you will always need three numbers!

This is no coincidence. The need for three numbers in the space of your room is intimately bound with the fact that your room is three dimensional. This means that there are three directions that do not effect each other -- these directions are independent. The dimension of a space measures the number of independent directions in that space. If you try to indicate the position of a point in a three-dimensional space with only two numbers, you are fixing only two of these independent directions. This leaves no information about the third, and the position of the point can be anywhere along that third direction. This means that in a three dimensional space we will need three numbers.

What about in the two dimensional space of this paper? Let's start over again in the context of this example. How would specify the position of the dot in the following paper space?

The most obvious way is to say that it is such a distance from the top of the page and such a distance from the left side. Or it could have been from the bottom and from the right side, or any combination of these. The main thing to notice here is that two numbers do it. There are other ways to specify the position of that point, and you may wish to amuse yourself by trying to picture them. All of them will take at least two numbers to do the job (of course, you can invent ways that take more than two numbers, by using each dimension more than once). This is because the space of the paper is two dimensional.

When you are specifying the position of a point in a way that uses the smallest number of numbers, these numbers are called the **coordinates** of the point. Thus in the space of the paper you will need two coordinates to specify a position. In the space of your room you will need three coordinates to specify the position of a point.

How does the motion of objects in space appear in terms of coordinates? Very simple! The coordinates of something that is moving in a space change as the something moves. Thus motion is simply the changing of coordinates.

Coordinates are tremendously important if you are going to attempt to make a precise description of nature. Physicists wish to describe objects in the universe, measure their motion and by so doing come to some understanding of how the universe is put together. In order to do this, they must use coordinates to make precise, simple statements about what the objects in the universe are doing. Without this, it would be very hard to see patterns in the behavior of objects, and without patterns it would be very hard to understand how the universe works.

But wait! How can coordinates be so important? After all, the coordinates we use to describe our spaces are entirely made up by us, and we could as easily as not have made up another set of coordinates for our description! This is a very important point to understand. We impose our own coordinate systems on nature. This does not mean that there is no independent nature out there that we are observing (ignoring for now the current controversies around quantum mechanics). It does mean that if we measure something, then for it to be actually part of nature, as opposed to some artifact of our measuring process, we had better find that this measurement has the same value in all coordinate systems. It is a wonderful and somewhat surprising fact that there are things in nature that satisfy this. Relativity theory is about finding those aspects of spacetime that are independent of what coordinates we use to measure them.