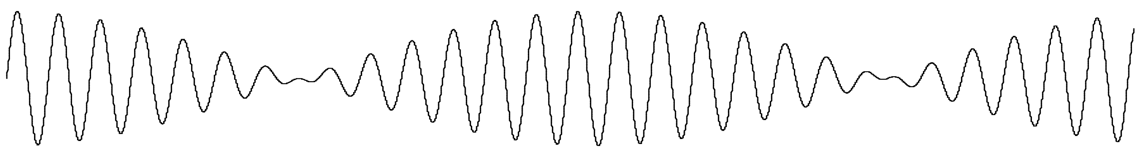
More Stuff About Waves and Particles

Last time, we learned that waves have a lot to do with particles. We learned that the wavelength of a wave determines the particle's state of motion and the strength of the wave determines the particle's possible position. However, a wave having a well defined wavelength is somehow incompatible with that wave being localized in space. If the wave is localized, it is not clear what you mean by wavelength, because the wavelength of a wave is the distance between **exactly** repeating points on the wave and if a wave is localized it cannot repeat over a great distance. Thus the state of motion and the position of a particle corresponding to that wave are somehow incompatible. This is known as the **Heisenberg uncertainty principle**, and is now a well-established principle in physics.

Adding waves to make new waves

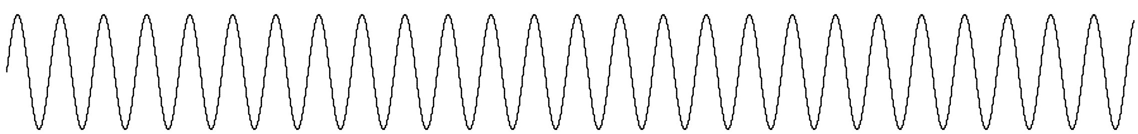
In figuring out what this means it now becomes very important for us to understand the relationship between a particle and it's associated wave. The first thought (due to Einstein and DeBroglie) was that the wave somehow "guided" the particle in a way that was not understood. To understand how this idea did not work out we need to know some more things about waves.

To motivate the following discussion, consider the following wave:

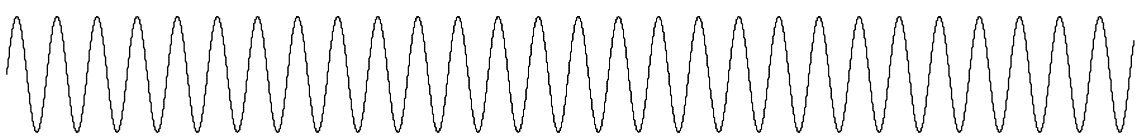


(This is actually a part of the wave that repeats forever.)

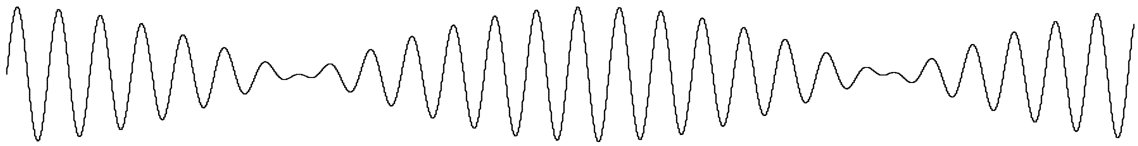
This is a rather complicated wave. I know, however, that it is really the sum of two simple sine waves (because this is how I drew it):



plus



equals



The two simple sine waves on top are of the same strength and slightly different wavelengths. The complicated wave at the bottom is the result of adding the two simple sine waves together. We can say that, in some sense, the complicated wave is made out of the two simple sine waves. Where the two sine waves both have wiggles in the same direction, the sum of the two waves is a wiggle twice as large in the same direction. Where the two sine waves have wiggles in opposite directions, they add up to no wiggle at all. Stare at the above figure for a while and see if you can see this.

We call the process of adding two waves to make a new wave **interference**. We say that the two waves interfere with each other to make a new wave when they add together as in the above example. The really fun thing about waves is that **any wave, no matter how complicated, may be expressed as a (possibly infinite) sum of simple sine waves.**

Now how is one to interpret interference in the context of the quantum rules? If the complicated wave were the quantum mechanical wave associated to a particle, what could we say about the particle's state of motion? If we take the quantum rules literally, then we must say that the particle somehow has both states of motion corresponding to the two simple sine waves that make up the complicated wave. This is an extremely radical statement, which anticipates our discussion of measurement.

To say that a particle can have two states of motion (in this case two velocities) at once seems completely against our intuition. (We will find later that according to the quantum theory of measurement, we only see one state of motion when we make a measurement, even though quantum mechanics says that there may be many.) Yet this is what we must believe if we assume the quantum rules associating waves to particles.

So now we are really up against the wall of either trusting our intuition or believing quantum mechanics. In this position, the physicist always turns to experiment. In 1927 Davisson and Germer performed an experiment in which a beam of electrons showed interference. In fact, the interference observed exactly matched that expected from the quantum mechanical wave associated with the electrons! They did not, however, observe interference in the wave. **They observed interference in the particles!**

This result could not possibly be stranger. It was not that an individual electron showed interference, yet when you looked at where many electrons went in this experiment, you found that they fell exactly where the **interfered** quantum wave for the electrons had high strength.

To Summarize: What we observe to be particles in nature seem to behave in a way determined by these mysterious waves. The waves determine the behavior of the particles in the following ways:

-If the particle's wave is a simple sine wave, then the particle has a velocity (actually momentum) determined by the wave's wavelength.

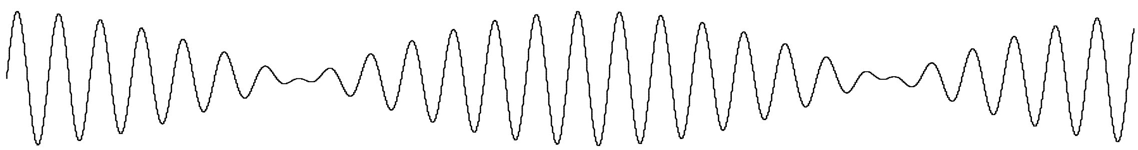
-If the particle's wave is just a spike at a point then the particle will be found at that point.

If the particle's wave has a form other than these two, then the particle will be observed to have one of several possible velocities (actually momenta) and may be found wherever the wave has a non-zero strength. The reason that there are several possible velocities for the particle is that no matter what the shape of the wave, that wave may be considered as the sum of many different simple sine waves each giving a possible exact velocity. The stronger the constituent sine wave, the more likely that the velocity given by that sine wave will be found.

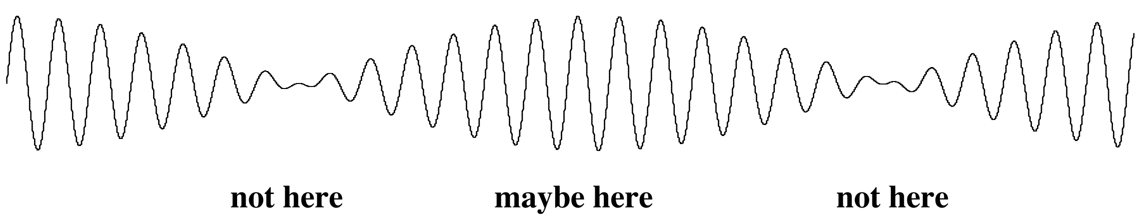
The act of measurement in quantum mechanics

The Collapse of the Quantum Wave

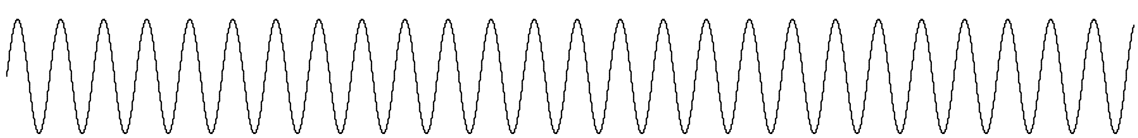
This puts us in a position to discuss just how one views the act of measurement in quantum mechanics. Consider the wave on the first page of this handout, which was the sum of two simple sine waves:

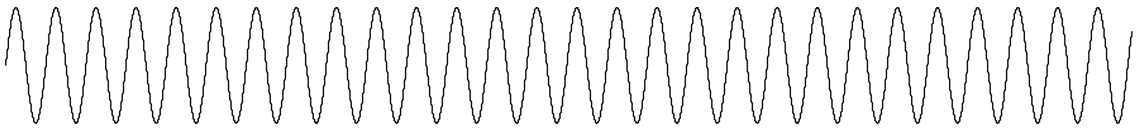


Let's say that this wave is the quantum wave associated to some particle. Then we can ask "Where will I find the particle and how fast will it be going?". We already know that the particle can be found wherever the strength of the particle is not zero, that is anywhere but the places where there is no wiggle:



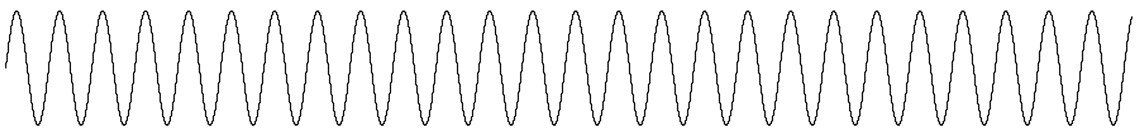
Now what can one say about the possible velocities of the particle that one can expect to observe? The particle's wave is exactly the sum of these two simple sine waves:





The two sine waves that make up this wave are of equal strength and slightly different wavelengths. This means that when we measure the velocity of the particle with the above complicated wave we are equally likely to find the particle with one of the two velocities given by the two constituent simple sine waves. Further, these two velocities are the only two possible velocities.

Now let's say that we have measured the particle's velocity and found it to be the velocity determined by the top of the two possible sine waves. **Now we know the particle's velocity exactly and therefore the particle's quantum wave is a simple sine wave:**



By the act of observation we changed the wave associated to the particle! The change of the wave due to the act of observation is called **the collapse of the wave function**. The wave, which before the observation was a sum of simple sine waves, collapsed due to the observation into **one** of the simple sine waves in that sum.

Now let's say that after we measured the particle's velocity exactly (which, by the way, made the particle's position totally indeterminate) we decide to measure the particle's position. Then after the measurement **we know the particle's position exactly and so it's wave looks like this:**

Now the wave has collapsed from a simple sine wave to a single pulse wave. Now this pulse wave is the sum of many more than two simple sine waves, so if we again measure the velocity of the particle the wave will collapse back to some simple sine wave, though we could not predict its wavelength. In other words the velocity has become indeterminate.

It is the collapse of the wave function that is at the heart of the controversies surrounding the interpretation of quantum mechanics. The central question in the interpretation of quantum mechanics is: **What is the quantum wave and how does it collapse?**

When phrased as above, it sounds like the quantum wave associated with a particle is somehow a measure of our knowledge of the particle and nothing more. But in last week's class we found that the quantum waves are directly observed in interference experiments and furthermore these waves explain the quantization of electron orbits in atoms. In other words we need to give some reality to the waves besides our knowledge in order to understand the physics we see.

What is that reality? The most popular views are summarized in the next section of this handout.

One thing I want to make clear at this point. The mathematics of quantum mechanics tells you what kind of quantum waves you will find in given situations. The mathematics **does not** seem to tell you about the collapse of the wave function or now it comes about. The collapse of the wave function has to be inserted into quantum mechanics as an **interpretation.**

Interpretations of the Wave Aspects of Matter

As we have seen, matter in the world appears to have wave aspects. The following is a short description of how different physicists view these wave aspects as parts of their physical world. There are several questions that come up:

Is the wave function a fundamental aspect of reality (as opposed to arising from some more fundamental particle system)?

Why is the probability of finding a particle proportional to the strength of the wave?

What is the relationship between particles and their associated waves?

Why does the wave function seem to collapse in the act of observation?

The **waveguide,** or **pilot wave theory** (DeBroglie, Einstein, Bohm): The wave is a fundamental aspect of reality which is separate from particles which are also fundamental. Thus there two kinds of objects in the world, waves and particles. The particles are in some yet to be understood way 'guided' by the waves and that is why the probability of finding a particle is given by the strength of the wave. It is easy to show that the equation for the quantum wave causes motion that looks like it is under the influence of a non-local force. In this case the particles behave classically, so they have well-defined positions and velocities at all times. The uncertainty of quantum mechanics arises from our lack of exact knowledge of the initial positions and velocities of the particles. This lack of knowledge is amplified by the quantum wave force as the particle moves. The wave is also controlled by the particles, so when we observe a particle we actually change that particle's associated wave. In this picture the wave does not actually collapse.

A vocal minority of physicist takes this point of view these days, and it was one of the first interpretations.

The **Copenhagen Interpretation** (Bohr, Heisenberg and a host of others): We just do not know what the wave associated to a particle is. We do, however, know how to predict observations from the wave function. Therefore let's just make those predictions in terms of things that we know (classical observables) and ask the above questions only during happy hour, not in the labs. This amounts to saying that observable quantities only make sense in the context of the observation. Put more strongly, it is improper to speak of, say, a particle's velocity unless you describe how you measure it. Since velocity is measured in a completely different way than position, it is no surprise that you cannot know the velocity and the position of a particle at the same time. In fact, it is not clear that it is meaningful to say that a particle has a velocity and a position at the same time, because you cannot even in principle measure them in the same experiment.

This is the popular point of view, in part because it allows physics to proceed.

Hidden variables (Bohm): The wave function is not fundamental. It arises from some averaging process over more classical particle properties that we do not yet know about. All the above questions are therefore misguided and we should be worrying about what these hidden, more fundamental particle properties are.

Due to the work of Bell and the observations by Aspect and others, we now know that in order to correctly describe the world, hidden variable theories must postulate influences that travel infinitely fast. According to relativity theory, such signals could perhaps enable one to send messages backwards in time thus destroying the notion of cause and effect. But it has been shown that such infinitely fast signals cannot be used to send messages without violating the requirement that the theory look like quantum mechanics. On the other hand, orthodox quantum theory correctly predicts all observations made so far without such problems, so hidden variable theories become somewhat less palatable.

Consciousness theories (Wigner): The world is built exactly as quantum theory says it is, and the wave function is the fundamental object which when is itself quantized gives rise to all particle phenomena. The collapse of the wave function is caused by the interaction of consciousness with the physical system described by the wave. Thus the world is not as consciousness sees it until consciousness sees it. The probability interpretation somehow arises out of the interaction of the world with consciousness (in a way that has yet to be understood).

While this point of view has its charms as a philosophical view of consciousness observing the world, its construction as a physical theory is highly problematical. Not a widely held view.

Many Worlds (Everett, Dewitt, Graham, originally Wheeler but Wheeler has moved away from this lately): The world is built exactly as quantum theory says it is, and the wave function is the fundamental object which when is itself quantized gives rise to all particle phenomena. There is in fact no mystery generated by the above questions. All you have to do is take the mathematical structures of quantum theory as literal representations of how the world is structured. Then you discover that the wave function never collapses and the world continually splits into many worlds in which all the quantum allowed possibilities occur. However, no one branch of the split can ever know of any others and so it feels like the wave function collapsed. Adherents of the many worlds interpretation originally claimed that the probability interpretation comes naturally from the mathematics of quantum theory, but this claim has not withstood scrutiny.

It is accepted that this point of view is technically correct and consistent, but due to the stated splitting of universes not many physicists actually believe it. Also, because the predictions of the many worlds interpretation exactly matches the predictions of quantum mechanics many physicists do not feel a need to choose it so they adhere to the Copenhagen view. I feel a need to stress, however, that the many worlds point of view is not strictly an interpretation--it is merely an observation of what the mathematics of quantum mechanics says, in particular that if you are some part of a quantum wave the world would appear to you exactly as it does.

Yet quantum mechanics is a completely deterministic theory. It determines the behavior of the abstract waves in the abstract space that quantum mechanics says is the reality. Since particles are not in the underlying reality of quantum mechanics it is no surprise that the properties of particles do not act like the elements of an objective reality.